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Farm Size Growth in Germany
The Measure Matters

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21st Pacioli Workshop
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Structure of the presentation

1. **Motivation** → *exploring structural change*

2. **Alternative measures** → *to weight or not …*

3. **Empirical analysis** → *results from Germany*

4. **Estimation vs. calculation** → *level of precision*

5. **Conclusions** → *… and further working steps*

What does the literature tell us?

- *Unfortunately both of the more common measures … (the arithmetic mean … and the median) are sensitive to the total number of observations …* (Lund and Price, 1998, p. 102).

- *[The mean or median] … are extremely sensitive to the definition of a farm, which has changed implicitly or explicitly over time …* (Roberts and Key, 2008, p. 628).

- *An alternative approach […] is to give less emphasis to the total number of holdings and to pay more attention to the distribution of the land between holdings …* (Britton, 1950, p. 191).

- *[The weighted median] … is less arbitrary than the mean or median since a change in the number of small farms cannot alter it if industry output is not appreciably affected* (Weiss, 1963, p. 74).

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Measuring farm size - an illustrative example

900 hectares over 9 farms
→ equal distribution

100	100	100
100	100	100
100	100	100

Mean (μ)	**100**
Median (m)	**100**
Hectare-weighted mean (μ_{hw})	**100**
Hectare-weighted median (m_{hw})	**100**

Merging and splitting of farms
→ higher concentration

20		
40	140	120
60		
80	160	180
100		

Mean (μ)	**100**
Median (m)	**100**
Hectare-weighted mean (μ_{hw})	**126.7**
Hectare-weighted median (m_{hw})	**124.3**

Source: Modified from Key and Roberts, 2007, p. 10

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Measures of farm size (I)

Mean (μ)

$$\mu = \frac{1}{n} \times \sum_{i=1}^{n} uaa_i$$

utilised agricultural area

Hectare-weighted mean (μ$_{hw}$)

$$\mu_{hw} = \sum_{i=1}^{n} \left(uaa_i \times \frac{uaa_i}{\sum_{i=1}^{n} uaa_i} \right)$$

$$= \sum_{i=1}^{n} \left(\frac{uaa_i{}^2}{\sum_{i=1}^{n} uaa_i} \right)$$

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Measures of farm size (I)

Mean (μ)

$$\mu = \frac{1}{n} \times \sum_{i=1}^{n} uaa_i$$

utilised agricultural area

$$\mu = \sum_{i=1}^{n} \left(uaa_i \times \frac{wf_i}{\sum_{i=1}^{n} wf_i} \right) = \frac{\sum_{i=1}^{n}(uaa_i \times wf)_i}{\sum_{i=1}^{n} wf_i}$$

with FADN weighting factors (*wf$_i$*)

Hectare-weighted mean (μ$_{hw}$)

$$\mu_{hw} = \sum_{i=1}^{n} \left(uaa_i \times \frac{uaa_i}{\sum_{i=1}^{n} uaa_i} \right)$$

$$\mu_{hw} = \sum_{i=1}^{n} \left(uaa_i \times \frac{uaa_i \times wf_i}{\sum_{i=1}^{n}(uaa_i \times wf_i)} \right)$$

$$= \sum_{i=1}^{n} \left(\frac{uaa_i{}^2}{\sum_{i=1}^{n} uaa_i} \right)$$

$$= \sum_{i=1}^{n} \left(\frac{uaa_i{}^2 \times wf_i}{\sum_{i=1}^{n}(uaa_i \times wf_i)} \right)$$

with FADN weighting factors (*wf$_i$*)

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Measures of farm size (II)

Median (m)

$$m = uaa_{\frac{n+1}{2}} \quad \text{or}$$

$$m = \frac{1}{2}\left(uaa_{\frac{n}{2}} + uaa_{\frac{n}{2}+1}\right)^{*}$$

Hectare-weighted median(m_hw)

$$m = uaa_{\gamma} \ ;$$

$$\sum_{i=1}^{\gamma-1} uaa_i < \frac{1}{2}\sum_{i=1}^{n} uaa_i \geq \sum_{i=\gamma}^{n} uaa_i$$

* For odd or even numbers, respectively.

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Measures of farm size (II)

Median (m)

$$m = uaa_{\frac{n+1}{2}} \quad \text{or}$$

$$m = \frac{1}{2}\left(uaa_{\frac{n}{2}} + uaa_{\frac{n}{2}+1}\right)^{*}$$

$$m = uaa_{\gamma} \quad \text{with} \quad \sum_{i=1}^{\gamma-1} wf_i < \frac{1}{2}\sum_{i=1}^{n} wf_i \geq \sum_{i=\gamma}^{n} wf_i$$

with FADN weighting factors (*wf_i*)

Hectare-weighted median(m_hw)

$$m = uaa_{\gamma} \ ;$$

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$$m = uaa_{\gamma} \quad \text{with} \quad \sum_{i=1}^{\gamma-1}(wf_i \times uaa_i)$$

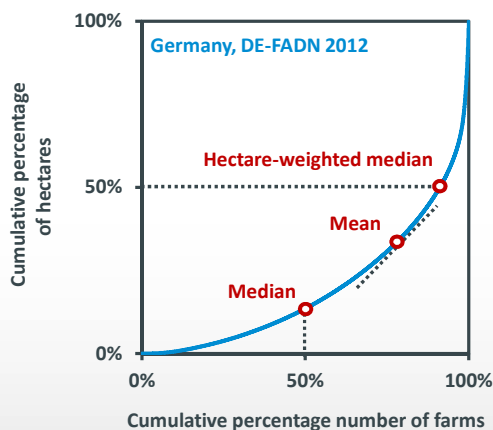$$< \frac{1}{2}\sum_{i=1}^{n}(wf_i \times uaa_i) \geq \sum_{i=\gamma}^{n}(wf_i \times uaa_i)$$

with FADN weighting factors (*wf_i*)

* For odd or even numbers, respectively.

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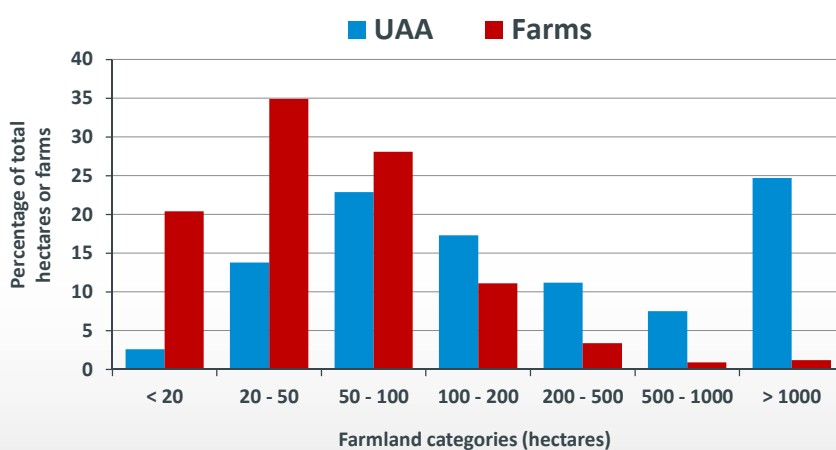
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Measuring farm size –
median, weighted median and mean

Germany, DE-FADN 2012

Cumulative percentage of hectares

100%

Hectare-weighted median

50%

Mean

Median

0%

0%   50%   100%

Cumulative percentage number of farms

Source: Modified from Lund and Price, 2007, p. 5

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Percentage distribution of farmland
and number of farms, DE-FADN 2012

■ UAA   ■ Farms

Percentage of total hectares or farms

Farmland categories (hectares): < 20, 20 - 50, 50 - 100, 100 - 200, 200 - 500, 500 - 1000, > 1000

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Alternative measures of average farm size for Germany, DE-FADN

Germany, DE-FADN				
	2005	**2012**	**difference**	**%-difference**
Mean	64.5	86.0	**21.5**	**33.4**
Median	34.1	44.3	**10.2**	**29.9**
Hectare-weighted mean	552.4	629.8	**77.4**	**14.0**
Hectare-weighted median	106.4	145.4	**39.0**	**36.7**
Germany, old Länder, DE-FADN				
Hectare-weighted mean	80.4	99.7	**19.3**	**24.0**
Hectare-weighted median	64.3	75.7	**11.4**	**17.7**

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Change of the percentage distribution from 2005 to 2012, DE-FADN 2005
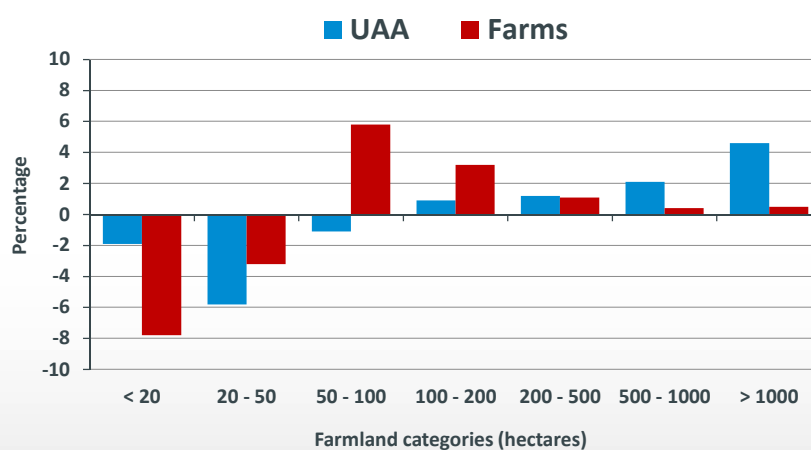
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Sensitivity of the measures – exclusion of farms
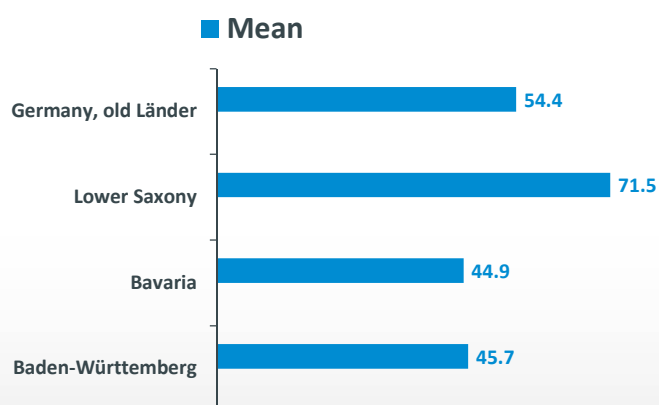
Germany, DE-FADN 2012

	All farms	Farms > 5 hectares	%-difference
Number of farms	192 758	182 525	-5.3
Total area (hectare)	16 583 382	16 562 941	-0.1
Mean	86.0	90.7	5.5
Median	44.3	47.1	6.3
Hectare-weighted mean	629.8	630.5	0.1
Hectare-weighted median	145.4	145.6	0.1

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Mean farm size (hectares)
in various German regions, DE-FADN 2012

■ Mean

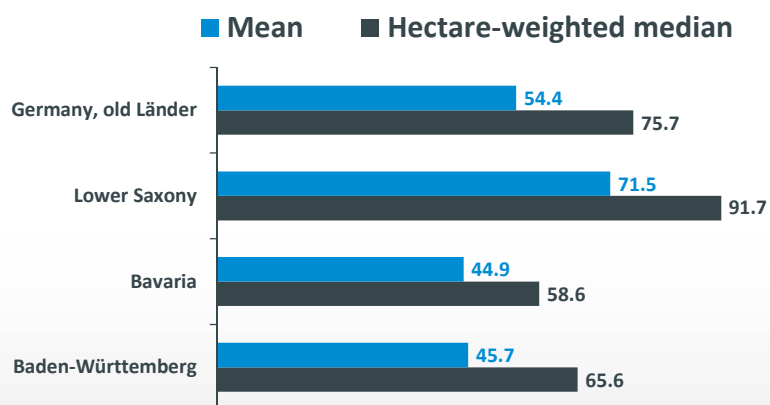|	
Germany, old Länder	54.4
Lower Saxony	71.5
Bavaria	44.9
Baden-Württemberg	45.7

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Mean and weighted median farm size (hectares) in various German regions, DE-FADN 2012
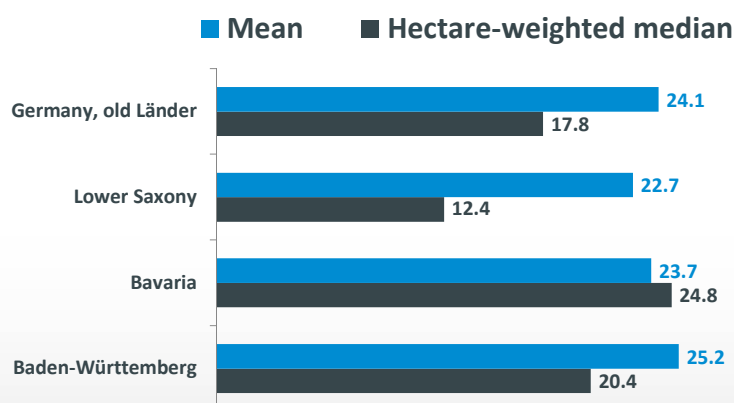
Mean ■ Hectare-weighted median

Germany, old Länder — 54.4 / 75.7
Lower Saxony — 71.5 / 91.7
Bavaria — 44.9 / 58.6
Baden-Württemberg — 45.7 / 65.6

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Percentage change of mean and weighted median farm size from 2005 to 2012, DE-FADN

Mean ■ Hectare-weighted median

Germany, old Länder — 24.1 / 17.8
Lower Saxony — 22.7 / 12.4
Bavaria — 23.7 / 24.8
Baden-Württemberg — 25.2 / 20.4

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Some notes on estimating (hectare-)weighted medians

- In order to calculate the (weighted) median, data on the level of individual farms is necessary (as is the case with FADN).
- In the absence of this data, the weighted median can be estimated by interpolation from percentage distributions.
  - Britton (1950) uses Lagrange's method of interpolation.
  - Lund and Price (1998) use cubic spline interpolation.
- Algebraically, the cumulative percentage numbers of hectares for the various size classes are used as *x* and *y* values …
  - … to find the polynom satisfying $F(x_i) = y_i$ and
  - … to compute the weighted median $m_{hw} = F(50)$.

Comparison of calculated and estimated hectare-weighted medians, DE-FADN 2010

Farmland categories (hectares)	Percentage of total	Cumulative percentage (inverse)
50 - 100	23.6	77.9
100 - 200	16.8	54.3
200 - 500	10.6	37.5
500 - 1000	6.8	26.9
> 1000	20.2	20.2

* Based on Lagrange's method of interpolation.

Comparison of calculated and estimated hectare-weighted medians, DE-FADN 2010

Farmland categories (hectares)	Percentage of total	Cumulative percentage (inverse)
50 - 100	23.6	77.9
100 - 200	16.8	54.3
200 - 500	10.6	37.5
500 - 1000	6.8	26.9
> 1000	20.2	20.2

Results for the weighted median
Estimation m_{hw} = 123.6 *
Calculation m_{hw} = 115.3

* Based on Lagrange's method of interpolation.

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Calculated and estimated hectare-weighted medians, DE-FADN and agricultural census

	Estimation* DE-FADN	Calculation DE-FADN	Estimation* (census)
Germany 2012	144.4	145.4	-
Lower Saxony 2012	91.7	91.7	-
Germany 2010	123.6	115.3	116.9
Germany 2007	132.5	110.9	109.1
Germany, old Länder 2010	63.4	67.6	73.7
Germany, old Länder 2007	64.4	69.0	68.1

* Based on Lagrange's method of interpolation.

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Differences between measures of average farm size between FADN and agricultural census data

	Hectare-weighted median		Mean	
	Calculation DE-FADN	Estimation* (census)	Calculation DE-FADN	Calculation (census)
Germany 2010	115.3	116.9	68.6	55.8
Germany 2007	110.9	109.1	67.6	52.2
Germany, old Länder 2010	67.6	73.7	46.2	40.6
Germany, old Länder 2007	69.0	68.1	47.0	37.9

* Based on Lagrange's method of interpolation.

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Conclusions (I)

- Hectare-weighted measures can complement analyses based on the more common (unweighted) mean and median
    - as they better reveal land concentration and are less sensitive to the inclusion or exclusion of small farms.
- However, hectare-weighted means are particularly sensitive to outliers at the upper end of the scale,
    - so that for distributions where large farms control most of the farmland results can be difficult to interpret.
- In the case of summary statistics instead of individual farm data, the weighted median can be estimated by interpolation.
    - However, results depend on the number of x and y values as well as the selected farm categories.

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Conclusions (II)

- Land concentration in Germany can be clearly demonstrated by using hectare-weighted measures.
- Given the distribution of farmland within Germany, i.e. importance of large farms in the new Länder, the hectare-weighted median is preferred to the hectare-weighted mean.
- Differences in average farm size between DE-FADN and agricultural census data are smaller if they are based on hectare-weighted medians than if they are based on (unweighted) means.

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Further working steps

… can examine
- systematically to what extent the alternative measures of farm size differ between FADN and agricultural census data.
- the trends in farm size growth using other measures like number of animals (e.g. milk cows) or standard output.
- and compare average farm size based on weighted medians across countries using exact calculation or estimation.
  (see efforts of the OECD Network for Farm Level Analysis)
- how results differ if alternative methods of interpolation are applied for the estimation of weighted medians.

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